

TALAT Lecture 2405

An Updating of TALAT Chapter 2400

Fatigue and Fracture in Aluminium Structures (Updated from the TAS project)

TAS



Leonardo da Vinci program
Training in Aluminium Alloy Structural Design

40 pages, 37 Figures

**Prepared by Prof. Dimitris Kosteas
Technische Universität München, Germany**

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2405 Fatigue and Fracture in Aluminium Structures (Updated from the TAS project)

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Abstract

Modern fatigue design standards require knowledge of fatigue data acquisition, documentation, representation and evaluation, i.e. the background of S-N design lines together with reliability statements. Fatigue design by testing is the next option. Finally, life estimation based on fracture mechanics and crack propagation material characteristics is introduced as a further option. This course material should be supplemented by the a) ENV 1999-2 (May 1998) on Fatigue Design of Aluminium Structures, b) the respective chapters in TALAT, c) the textbook „Metal Fatigue“ by N.E.Frost, K.J.Marsh and L.P.Pook in Oxford Engineering Science Series, 1974, and d) the book „Fatigue Testing and the Analysis of Results“ by W. Weibull in Pergamon Press, 1961.

A compact outline of the design procedures and the respective background information for the definition of S-N design lines for structural details for the Eurocode 9 / ENV 1999-2 has been published in STAHLBAU Spezial „Aluminium in Practice“, 67(1998), Ernst & Sohn/Wiley, Berlin, ISSN 0038-9145, pp 111-130. A comparison to design lines of other proposals and to actual fatigue data is also given here. Special attention is drawn to the design proposal of the ERAAS Fatigue Design document (1992).

This material has been utilized together with further definitions for classification of structural details to provide a proposal supported by the European Aluminium Association as a National Application Document, which may also be considered for introduction into the actual standard when this will be converted from an ENV to an EN. This material is included as a „supplement“ to this document.

2405.01. Fatigue Tests

2405.01.01 Experimental Investigations

The plain fatigue properties of a material will usually be determined on circular cross-section specimens, with careful surface finish. In practice though, few parts will have highly polished, undamaged surfaces and solely round cross-sections. Aluminium should be regarded in terms of a product rather and so test specimens will resemble or be parts of the actual shapes in practice, usually flat plate or sheet specimens. External corners may be mildly rounded, so that they do not give rise to early cracks, depending on the actual detail fatigue strength to be studied. In welded or otherwise notched specimens the crack initiation site will be rather clear from the beginning.

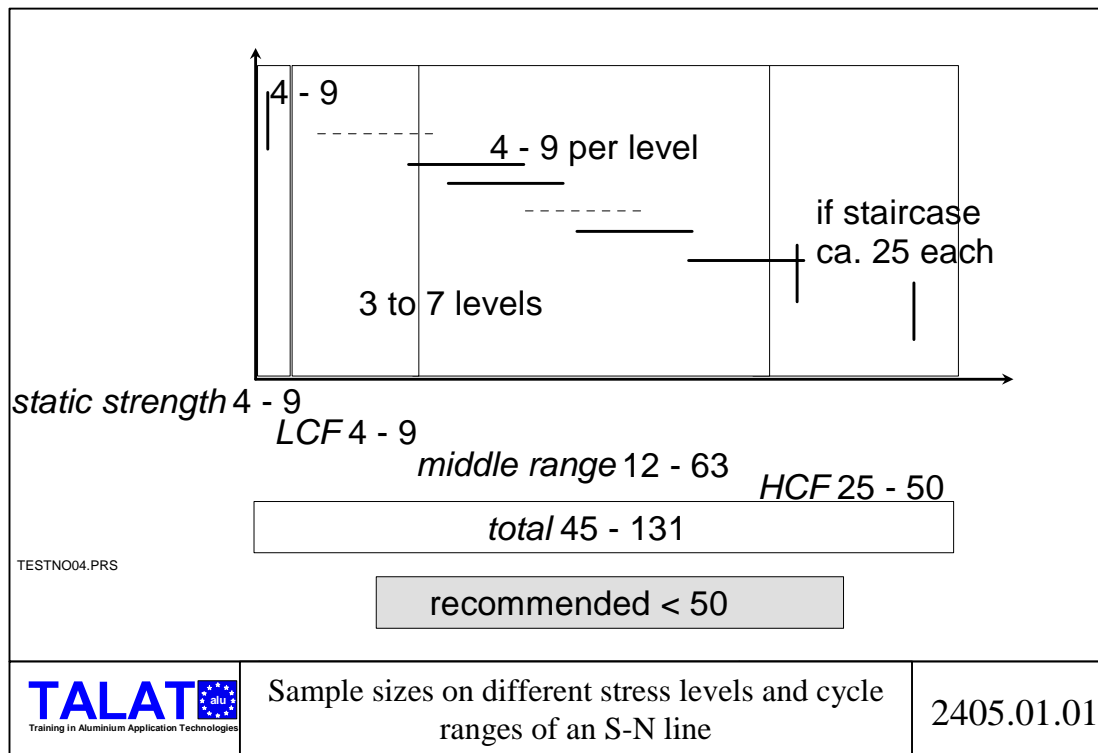
Because of the inherent variation of test piece parameters (material characteristics, alloy, product form, temper, geometrical parameters), further manufacturing parameters (joint type and procedure, post treatment, residual stresses), and environmental parameters (load and stress type, frequency, corrosion, temperature) a batch of nominally identical specimens is required. About the necessary number of specimens to be tested see below. In many cases a properly planned program, with subsequent statistical analysis of the results, may be required in order to be able to distinguish relevant differences in fatigue behavior. Guidance is provided in the textbooks mentioned in the abstract to this lecture for the preparation of test pieces, the testing program itself, and there is also ample literature on the subject in many national standards, especially in recommendations of the American Society for Testing and Materials. General information on fatigue testing machines is to be found in the mentioned textbooks, but information on newer models may be obtained directly from the manufacturers.

More difficult are general statements describing the preparation, set-up, and testing of full-size structural components. These components will usually be actual parts of the structure itself, often though will be formed and tested as H- or hollow-shape (double web, box) beams under pure bending (so called four-point bending) or combinations of bending, shear and axial load. Information may be obtained here from the respective reports of various laboratories performing such tests. As one example only, the report on the extensive aluminium beam program carried out in the eighties at the Technical University of Munich (contributing among other results to the background data for the European Recommendations and Standards) is mentioned: D. Kosteas and R. Ondra - Untersuchungen zum Schwingfestigkeitsverhalten geschweißter Verbindungen in Aluminium-Großbauteilen. Bericht Nr. 897On (AIF 7331), Laboratorium für den Konstruktiven Ingenieurbau der Techn. Univ. München, 1991.

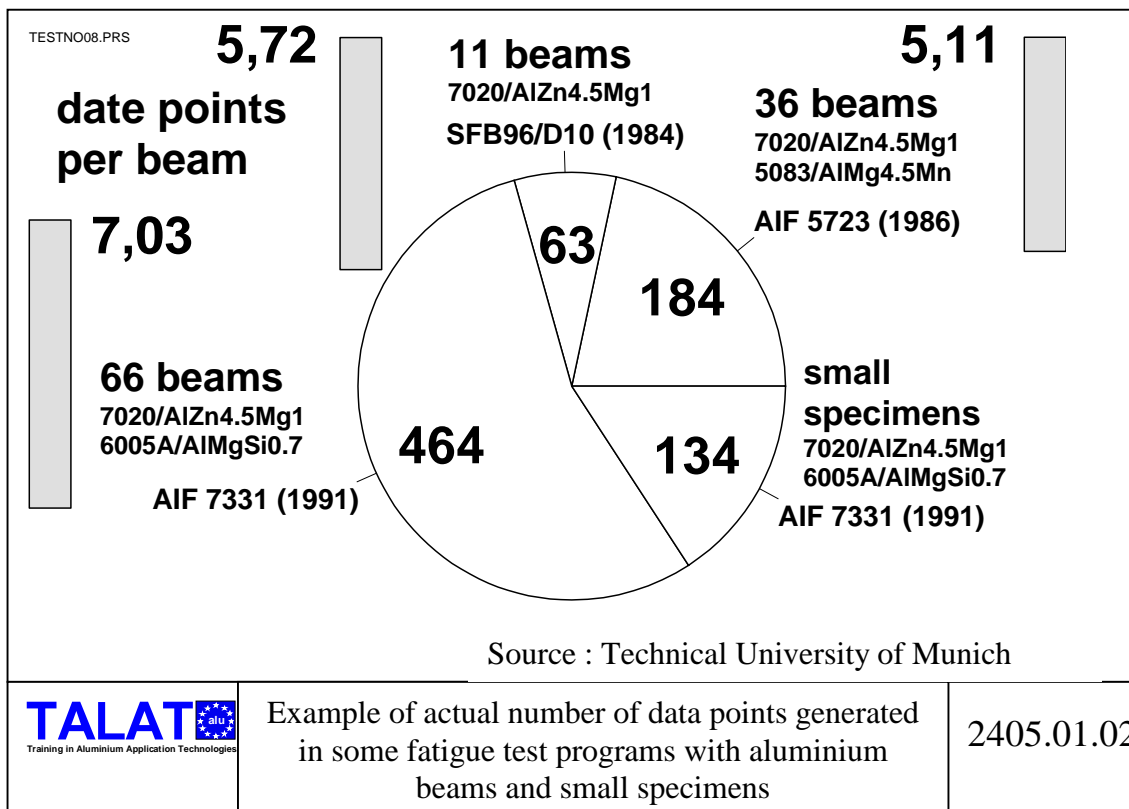
2405.01.02 Number of Specimens Required



There are numerous literature sources dealing with the theoretical statistical aspects of fatigue test data sample sizes required for a given probability and confidence limit. Practice will be characterized though by the given or possible (in manufacturing, economical or time limit terms) number of specimens. Certain general statements can be made based on the theoretical aspects as well as on experience from actual tests - see D. Kosteas, „Einfluß des Stichprobenumfangs bei der statistischen und regressionsanalytischen Auswertung von Schwingfestigkeitsversuchen, insbes. bei Schweißverbindungen aus AlZnMg1“ (Influence of sample size in the statistical and regressional analysis of fatigue test data). Aluminium,

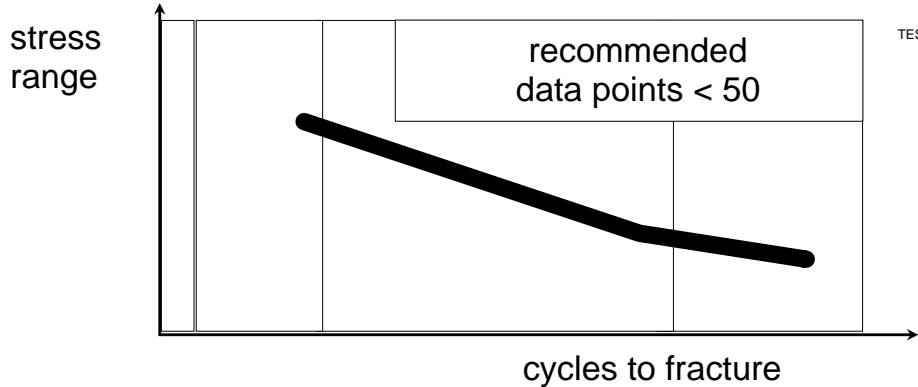

50(1974), 2, 165/170. The following figures recommend number of specimens for the estimation of an S-N line.



Recommended is a number of approximately 50 specimens if the whole range, including the constant amplitude fatigue limit, is to be established.



for 1 data point		TESTNO07.PRS
approximate cost		in DM in the time between 1980 + 1990
full-size components (beams) up to 3000		
small specimens		300
<i>influenced by detail type, weld, testing frequency or test duration, constant or variable loading, possible crack registration, data documentation and evaluation procedures</i>		
frequency / test duration for 2 million cycles		
roughly 2 Hz	full-size component	roughly 14 days
6 (to 10) Hz	small specimen	4 days
	Average cost and duration of test for typical small specimens and beams in aluminium.	2405.01.03

	TESTNO05.PRS
>100 000 DM from components < 20 000 DM from small specimens	
	Approximate cost for an S-N line.
2405.01.04	

2405.02 Fatigue Data Analysis and Evaluation

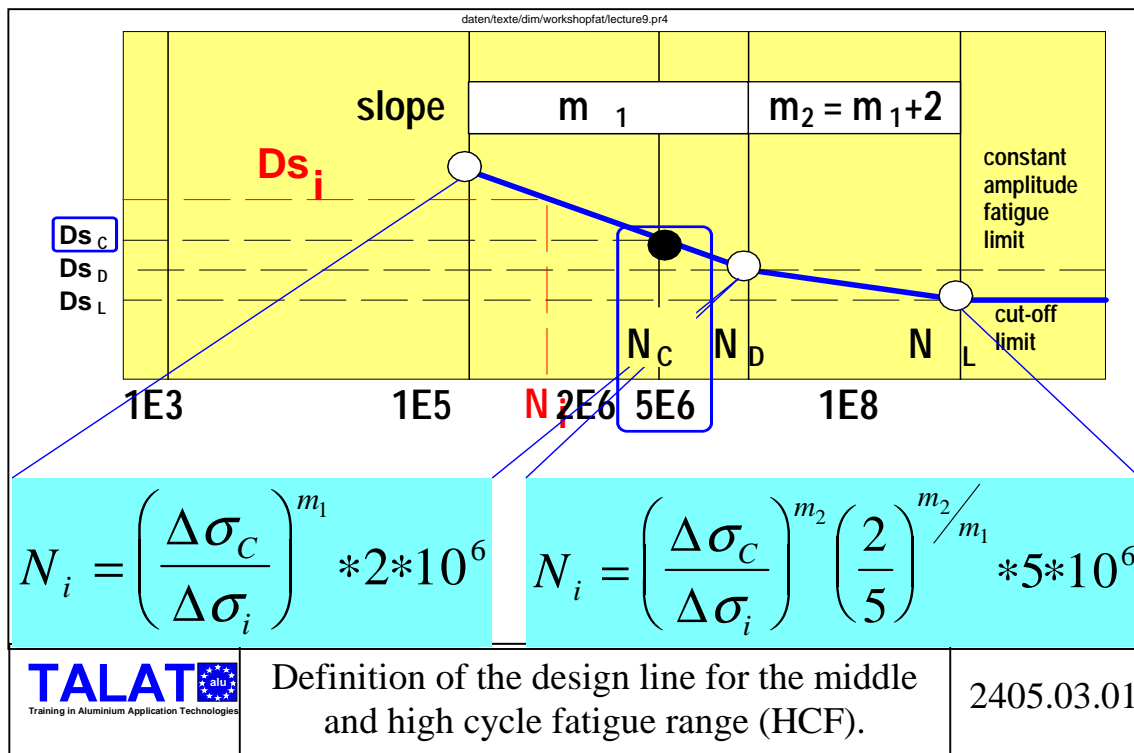
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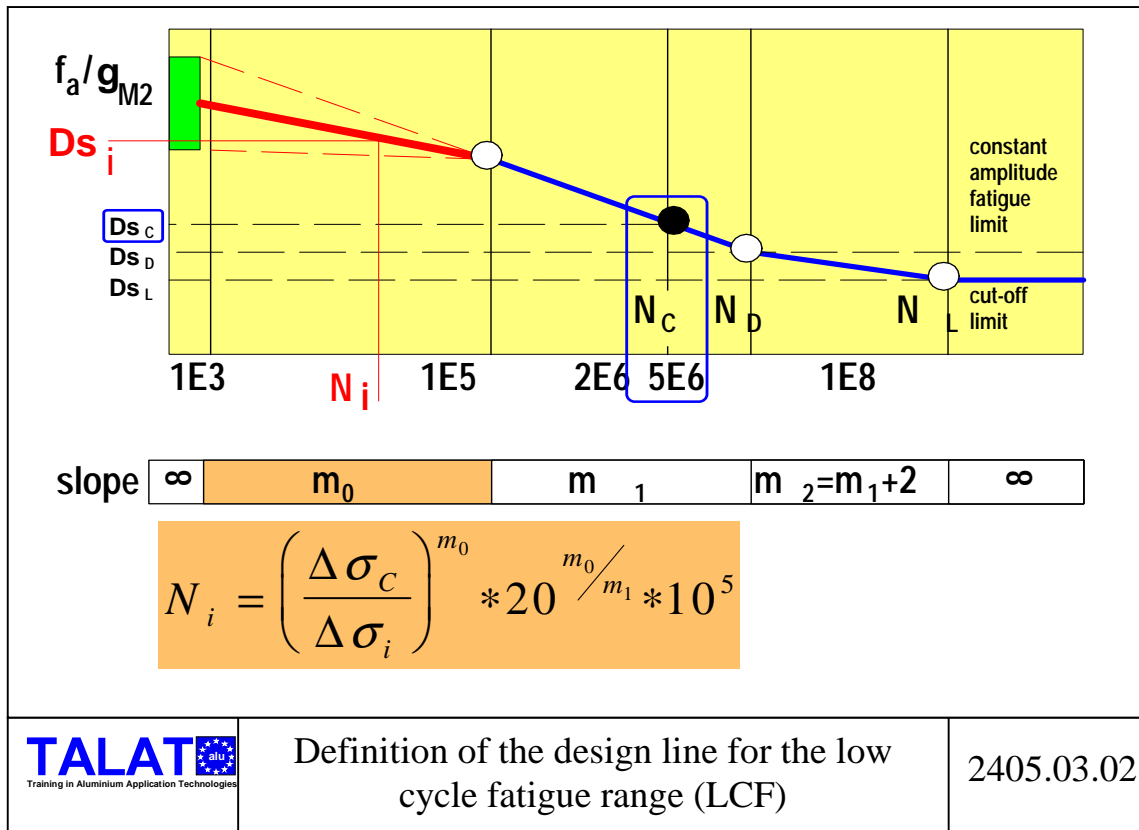
The following pages give a more detailed background of the analysis of data procedures, the common fatigue diagrams, and the linear or also, where appropriate, the non-linear probability-stress-cycles P-S-N curves as they will be utilized in design.

This chapter 2405.02 may be regarded as informative and it is not mandatory reading for understanding most applications mentioned in the following chapters. Important information on the P-S-N curves is summarized in the following chapter 2405.03 „Fatigue Design Line“.

2405.03 Fatigue Design Line

The fatigue design line or fatigue strength curve, as defined in the ENV 1999-2 (Eurocode 9 Fatigue Design) is a multiple slope, straight line in the double logarithmic stress-life diagram with a characteristic probability of survival, **Figure 2405.03.01** and **Figure 2405.03.02**. The respective partial safety factors are not included in the equations for the parts of the line since these are, generally, assumed equal to unity, as is explained in more detail in 2405.04.1.1. The characteristic value $\Delta\sigma_c$ is the reference value of fatigue strength at 2×10^6 cycles stated in N/mm^2 , depending on the category of the detail.





The design line represents according to the ENV 1999-2 a 2 standard deviation level below the mean line through experimental data. Theoretically (normal distribution assumed) this would correspond to a probability of survival of 97,7%. Practically, assuming an average sample size of six specimens per stress level, the mean minus 2 standard deviation level with a probability of survival of 97,7 % would have, though, only a confidence level of $\gamma = 0,5$.

The following **Figure 2405.03.03** states the k values - for the mean minus k standard deviation levels - at some characteristic probability percentiles and respective sample sizes together with the respective confidence levels.

sample size n	Probability of survival p_s								
	90			95			99		
	confidence level γ								
	0,50	0,90	0,95	0,50	0,90	0,95	0,50	0,90	0,95
3	1,498	4,258	6,158	1,939	5,310	7,655	2,765	7,340	10,55
6	1,360	2,494	3,006	1,750	3,091	3,707	2,483	4,242	5,062
12	1,316	1,966	2,210	1,691	2,448	2,736	2,395	3,371	3,747
25	1,297	1,702	1,838	1,666	2,132	2,292	2,357	2,952	3,158
(theoretical) Normal distribution	1,282			1,645			2,326		

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Values of k for estimations of mean minus k standard deviation levels

2405.03.03

2405.04 Safety and Reliability In Aluminium Design

This chapter is based on a paper by D. Kosteas and R. Ondra presented on the 6th INALCO Conference on Aluminium Weldments, April 3-5, 1995, in Cleveland, Ohio, USA. It has also been published as IIW Doc. No. XIII-1586-95.

Based on the elements of the safety concept as defined in current european recommendations an evaluation is undertaken for actual values with structural details in welded aluminium components under fatigue loading especially. Limit values of the safety index are indicated for conditions in practice.

2405.04.01 Safety Concept in Recommendations

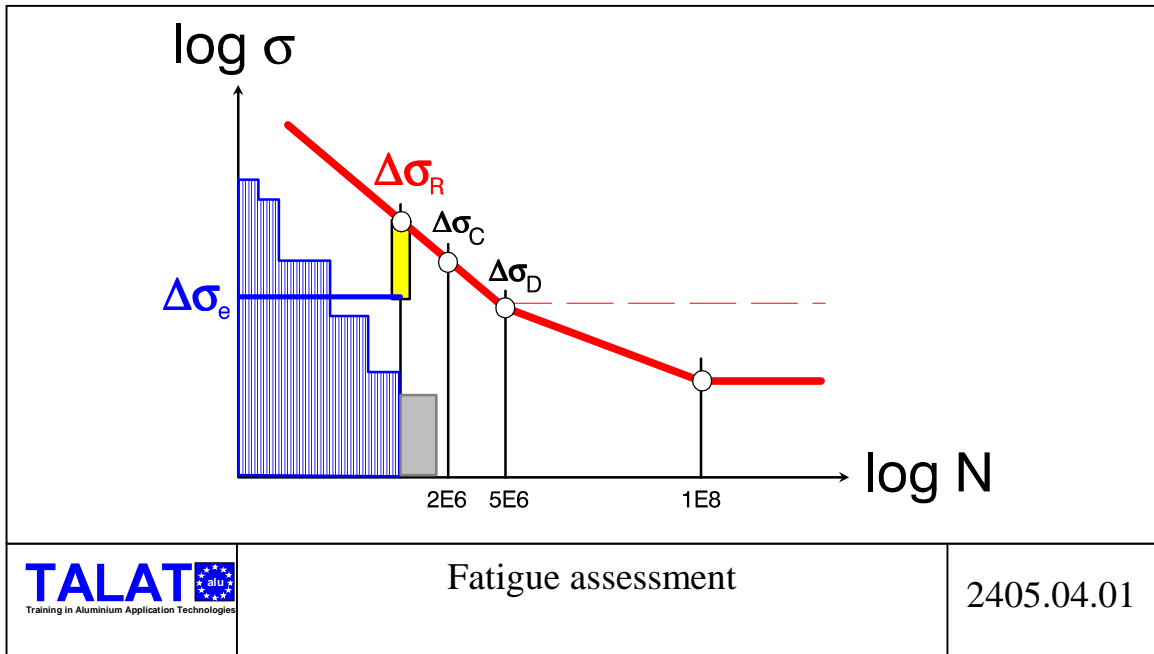
A safety concept on a semi-probabilistic basis, independent from the construction type, has been introduced in building codes and is expressed by a required value of the reliability index in current European design recommendations, as for example the Eurocodes [1] or recently released national codes [2]. There is a rather explicit definition for partial safety factors for actions (for example ranging from 1.00 to 1.50 and whether favorable or unfavorable effects, permanent or variable actions are considered [1]) and for resistance (generally 1.10 for resistance related to the yield strength - instability - or 1.25 for resistance related to the ultimate tensile strength [1]) to be applied to the static assessment of structures. In the case of fatigue assessment a harmonization of assumptions has to be introduced as there are a number of different suggestions for the actual values of the safety index or the partial safety factors to be applied.

In the case of fatigue the assessment of the structures or parts of it can be expressed generally by the comparison of an damage equivalent stress range $\Delta\sigma_e$ to the appropriate design stress value $\Delta\sigma_R$ for the corresponding structural detail, whereby the respective partial safety factors for loading γ_F and for resistance γ_M have to be taken into account, **Figure 2405.04.01**. The analytical expressions are ($\Delta\sigma_i$ and n_i for stresses higher than $\Delta\sigma_D$ at the part of the S-N line with slope m_1 and $\Delta\sigma_j$ and n_j for stresses lower than $\Delta\sigma_D$ at the part of the S-N line with slope m_2) for the equivalent stress range

$$\Delta\sigma_e = \left[\frac{\sum (n_i \Delta\sigma_i^{m_1}) + \Delta\sigma_D^{m_1 - m_2} \sum n_j \Delta\sigma_j^{m_2}}{\sum n_i + \sum n_j} \right]^{\frac{1}{m_1}}$$

and for the fatigue assessment itself

$$\gamma_F \Delta\sigma_e < \frac{\Delta\sigma_R}{\gamma_M}$$



Partial Safety Factors for Fatigue Loading

The European document ENV 1991 Eurocode 1: Basis of design and actions on structures incorporates appropriate values of γ_F . The Eurocode 3 for steel structures [1] states that the fatigue assessment procedure shall incorporate a partial safety factor γ_F for fatigue loading to cover uncertainties in estimating

- the applied load levels,
- the conversion into stresses and stress ranges,
- the equivalent constant amplitude stress range,
- the design life of the structure, and the evolution of the fatigue loading within the required design life of the structure.

Especially the last point may have a significant influence on the assessment procedure and should be taken into account accordingly. In our view the further statement of [1] that „unless otherwise stated in other parts of the code, or in the relevant loading standard, a value of $\gamma_F=1.00$ may be applied to the fatigue loading“ is somewhat irritating in this context.

The existing European Recommendations [3] do not go into any details whatsoever about the value of γ_F but state rather generally that „fatigue loadings must be defined for the assessment of the structure in accordance with its intended application“ and that „they should represent an upper bound estimate of the fluctuating loads to be experienced by the structure or the part during the full design life“.


The respective British document [4] places the decision upon the designing engineer but is more specific about the values recommended. The „nominal design life“ (the period in which the structure or component is required to perform safely) may be increased in certain circumstances by the „fatigue life factor“, producing thus the so-called „factored design life“.

The fatigue life factor $\gamma_L > 1$ (corresponding to the above-mentioned fatigue load/stress factor γ_F , the relationship expressed through $\gamma_L = N_2/N_1 = (\Delta\sigma_1/\Delta\sigma_2)^m = (1/\gamma_F)^m$) accounts for

- the possibility of increasing crack growth during the later stages of the life of the detail,
- the accuracy of the assumed loading spectrum,
- whether records of loading will be kept during the life of the detail
- the possibility of change of use of the structure in mid-life.

New aluminium standards, as the ENV 1999 Eurocode 9: Design of aluminium structures, follow the above general recommendations. Because of the fact of the variety of loading patterns, application fields and situations to be assessed in fatigue - especially in the case of aluminium structures covering diverse application areas like buildings or transportation - there is, consequently, a difficulty in defining some universal value. The ENV 1999 - Eurocode 9 - Part 2: Design in Fatigue states with more detail that „a partial safety factor on load intensity $\gamma_{FF} = 1,0$ may be assumed to provide an acceptable level of safety, where the fatigue loading has been derived in accordance with the requirements“, i.e. observing specific rules about the characteristics of load spectra and their realistic assessment over suitable sampling periods. Where fatigue loading has been based on other confidence limits than those stated, an acceptable level of safety may be assumed to be provided by applying the partial safety factors on loadings given in **Figure 2405.04.02**. The values k_F and k_N describe the multiples of standard deviations of the intensity and the number of cycles respectively of the design load spectrum. Only in case of $k_F = k_N = 2$, or in other words when the percentile to be used for the intensity and the percentile to be used for the number of cycles of the load spectrum are both calculated from the respective mean value plus two standard deviations, the partial safety factor for fatigue loading may be assumed to unity.

k_F	γ_{FF}	
	$k_N = 0$	$k_N = 2$
0	1,5	1,4
1	1,3	1,2
2	1,1	1,0

	Partial safety factors γ_{FF} for fatigue load intensity after ENV 1999 (EC 9)	2405.04.02
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Partial Safety Factors for Fatigue Strength

In the case of steel structures older national documents - as for instance the German DIN 15018 and DIN 4132 for cranes and crane bridges - had stated only a partial safety factor of 1.33 on material resistance calculated as the 90% probability of survival limit (without any further details about the distribution, sample size, confidence level, etc. though) in order to calculate allowable values of fatigue strength.


The Eurocode 3 for steel structures states that the design value of the fatigue strength shall be obtained by dividing by a partial safety factor γ_M . It covers the uncertainties of the effects of

- the size of the detail
- the dimensions, shape and proximity of the discontinuities,
- local stress concentrations due to welding uncertainties,
- variable welding processes and metallurgical effects.

Recommended values are based on the assumption of specific quality assurance procedures applied and relative to the consequences of failure, whereby either

- „fail-safe“ structural components may be assumed, with reduced consequences of failure, such that the local failure of one component does not result in failure of the structure or
- non „fail-safe“ structural components have to be assumed where local failure of one component leads rapidly to failure of the structure.

Inspection and access	Component assumed	
	„fail-safe“	non „fail-safe“
Periodic inspection and maintenance. Accessible joint detail	1.00	1.25
Periodic inspection and maintenance. Poor accessibility	1.15	1.35

	Partial safety factors γ_M for fatigue strength after ENV 1993 [1]	2405.04.03
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The case of no periodic inspection is apparently not considered. There is also an option of adjustment of γ_M factors in cases where values of γ_F other than 1.00 are applied.

The British code [4] again states that „the designer may wish to apply a fatigue material factor $\gamma_M > 1.00$ and its choice could be influenced by a) the need for the detail to exist in a very hostile environment, and b) whether failure of the detail will result in failure of the entire structure, or whether alternative load paths exist“.

A factor $\gamma_M = 1.00$ is understood in the European Recommendations [3] for aluminium structures. There is only an indirect statement that „the design and fabrication of details should, as far as it is practical, allow for a) pre-service inspections in order to satisfy quality assurance requirements, b) in-service inspections, and c) detection of fatigue cracking. A further indirect reference is made in the provisions referring to acceptance testing where the criterion for acceptance depends upon whether the structure is required to give a safe-life performance or a damage tolerant performance. In the first case the design life is adjusted through a fatigue test factor dependent upon the effective number of test results. In the second case fracture mechanics methods lead to inspection procedures, whereby detectability of

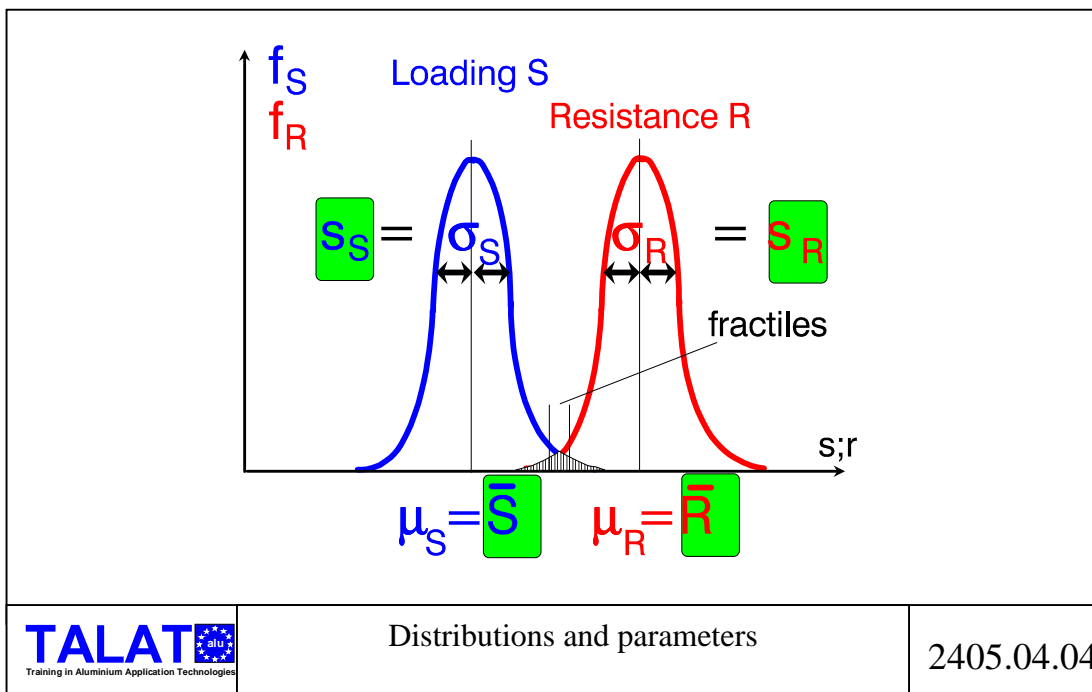
cracks, rate of crack growth, critical crack length considerations, implications for the residual safety of the structure and the cost of repair play a role.

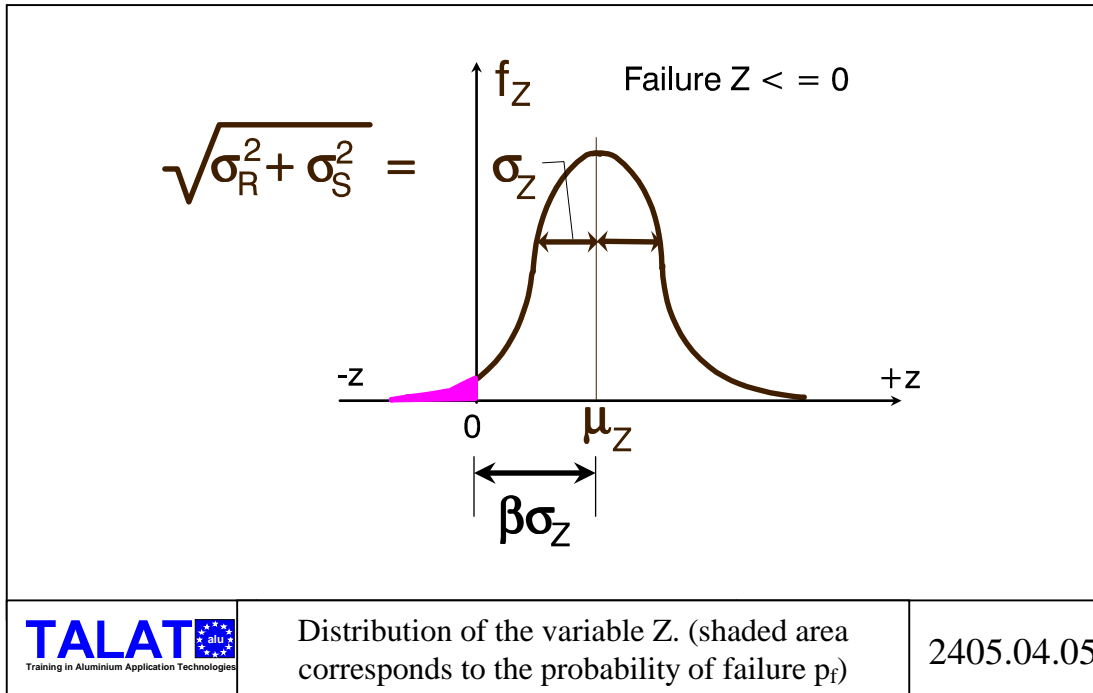
The Eurocode 9 document ENV 1999 defines for the partial safety factor on fatigue strength a value of $\gamma_{Mf} = 1,0$ (except for adhesively bonded joints, which should be based on testing or the safety factor will reach a higher value, approximately between 3 and 5).

As indicated the decision about the actual value depends upon specific considerations of the structure, its joints, the attained quality and reliability in manufacturing, its purpose, its degree of redundancy, its environment. And after all it is the combined influence of both partial safety factors that characterizes the actual safety margin. The following information on the safety index β and the relation to the partial safety factors γ_F and γ_M , as well as the relationship to statistical parameters of the loading and resistance distributions (referring to the manufacturing characteristics and quality classification of a structural detail), demonstrate the actual situation for aluminium structural components and helps towards the definition of appropriate values.

2405.04.02 Safety Index and Partial Safety Factors

Assuming normal distributions for loading (stresses) and fatigue strength along with the definitions in **Figure 2405.04.04** it follows that in the limit state definition $R-S=0$ or in the actual design assessment on the basis of a safety margin $Z=R-S>0$ the variable Z itself will be normally distributed. In this case the failure situation (when $Z<0$) can be defined according to **Figure 2405.04.05**.





The corresponding mathematical expressions are for the distributions of load or resistance

$$f_R(r) = \frac{1}{\sqrt{2\pi}\sigma_R} e^{-\frac{1}{2}\left(\frac{r-\mu_R}{\sigma_R}\right)^2} \quad f_S(s) = \frac{1}{\sqrt{2\pi}\sigma_S} e^{-\frac{1}{2}\left(\frac{s-\mu_S}{\sigma_S}\right)^2}$$

the coefficients of variance

$$\rho_R = \frac{\sigma_R}{\mu_R} \quad ; \quad \rho_S = \frac{\sigma_S}{\mu_S}$$

and the percentiles

$$r = \mu_R - k_R \sigma_R = \mu_R (1 - k_R \rho_R)$$

$$s = \mu_S + k_S \sigma_S = \mu_S (1 + k_S \rho_S)$$

where the coefficients k account for scatter according to sample size.

The probability of failure may then - with a transformation variable $U = (Z - \mu_Z) / \sigma_Z$ - be expressed as

$$p_f = p(z \leq 0) = \int_{-\infty}^0 f_Z(z) dz = p\left(u \leq -\frac{\mu_Z}{\sigma_Z}\right) = \int_{-\infty}^{-\frac{\mu_Z}{\sigma_Z}} f_U(u) du = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{\mu_Z}{\sigma_Z}} e^{-\frac{1}{2}u^2} du = \Phi\left(-\frac{\mu_Z}{\sigma_Z}\right)$$

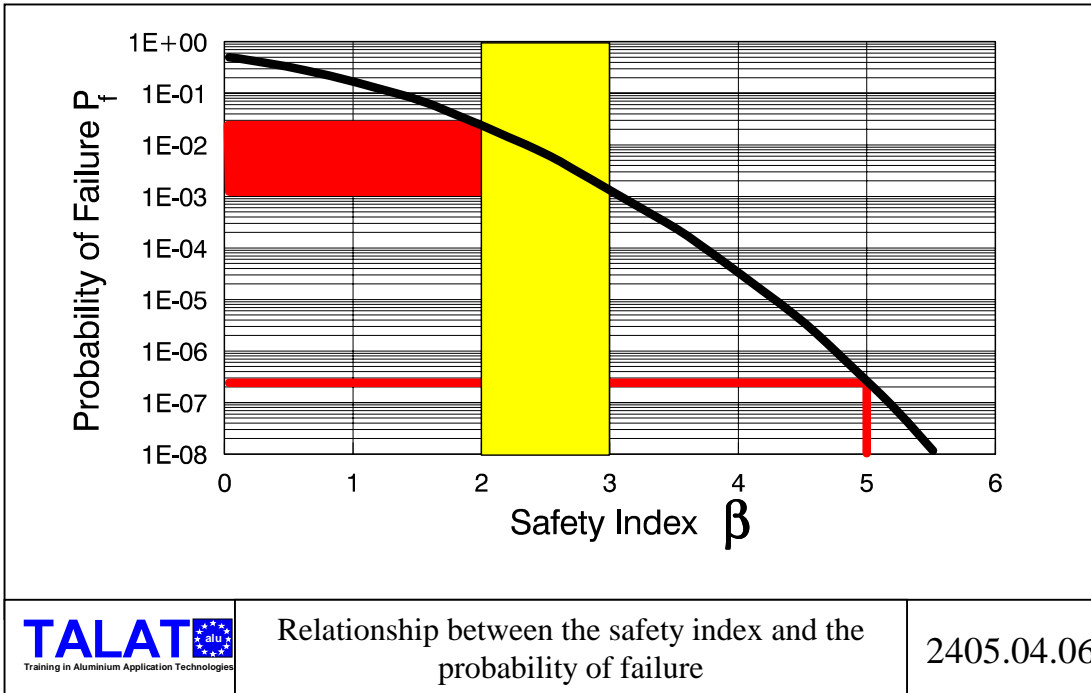
that means it has been transformed to a standardized normal distribution with a mean equal to 0 and a standard deviation equal to 1 and a relation may be established between the probability of failure p_f and the nominal safety factor (or the so-called global safety factor). So according also to the definition in **Figure 2405.04.04** we have

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta)$$

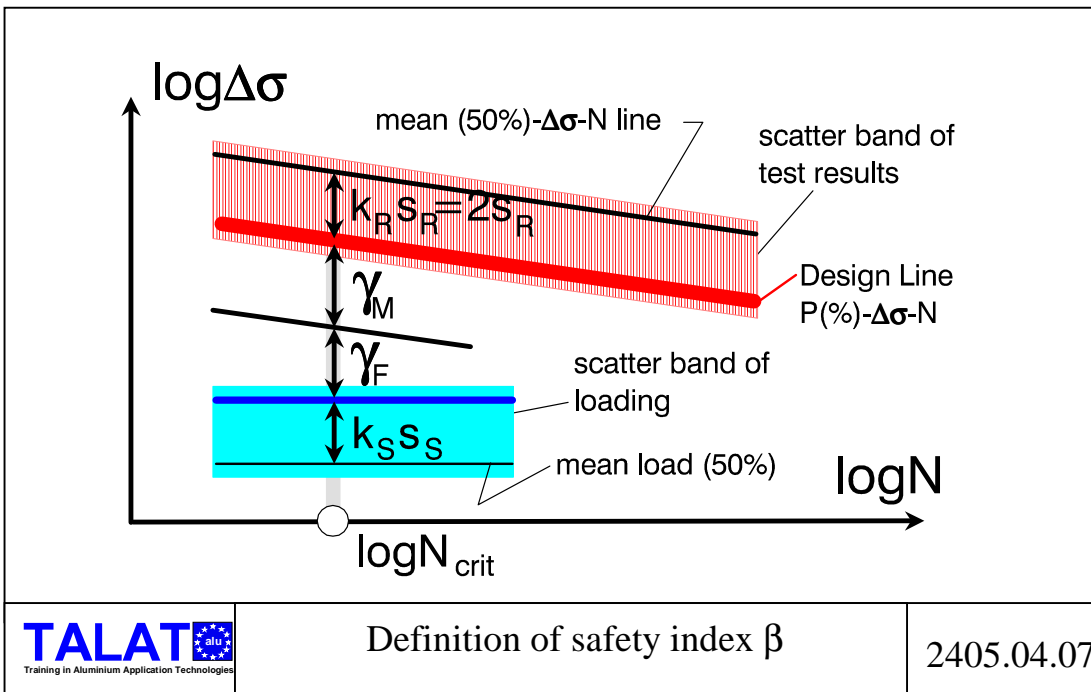
β being the safety index, corresponding to the inverse coefficient of variation of the quantity Z

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$


For the relationship $p_f - \beta$ see the following **Figure 2405.04.06**.



Following the above definitions we express the safety index as shown in **Figure 2405.04.07** and - **Figure 2405.04.08**.



$$\beta = \frac{k_S s_S + \log \gamma_F + \log \gamma_M + k_R s_R}{\sqrt{s_S^2 + s_R^2}}$$

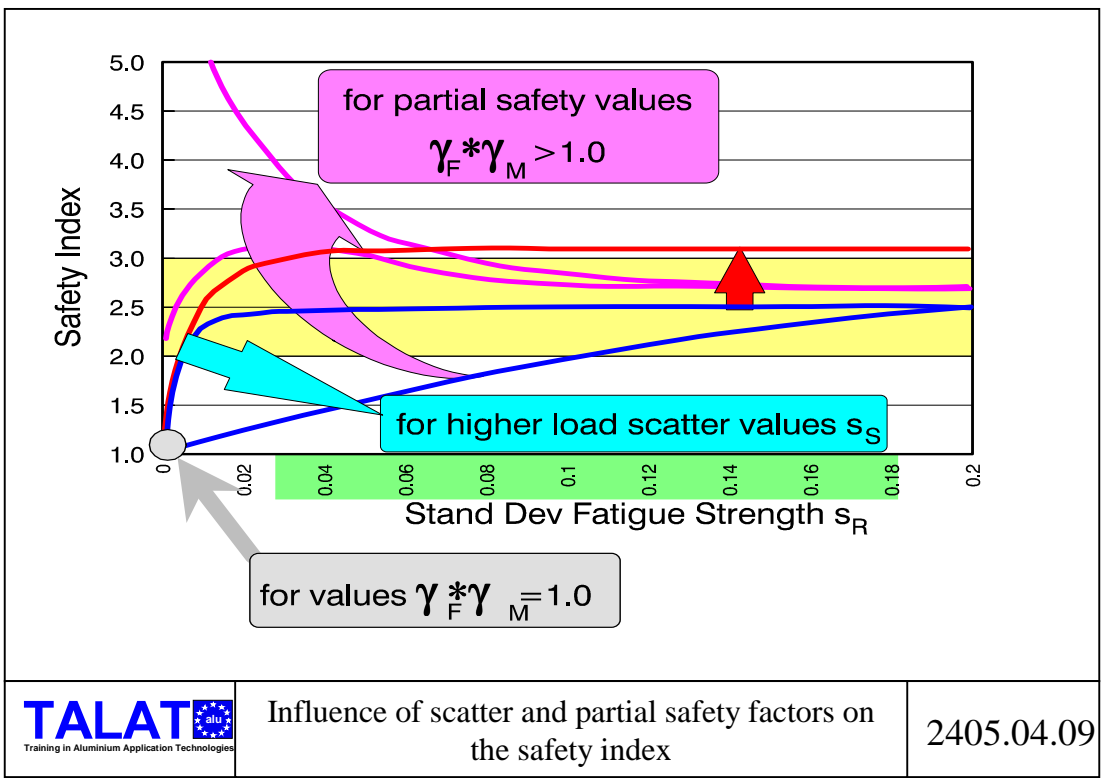
	Expression for β in practice	2405.04.08
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$k_S=1.0$ and $s_S=0.3$ as suggested in Background to Annex 11 of the Fatigue Rules / CEN. It should be noted here that this value for $k_S = k_F$ is not as high as demanded above, Table 2, i.e. equal to 2,0 so that a $\gamma_{FF} = 1,0$ may be assumed. Further considerations shall be needed in this. $k_R=2.0$ assumed in CEN/EC9 or ERAAS [3]

$\log \gamma_F + \log \gamma_M$ corresponds to „safety“ $\gamma_F * \gamma_M$

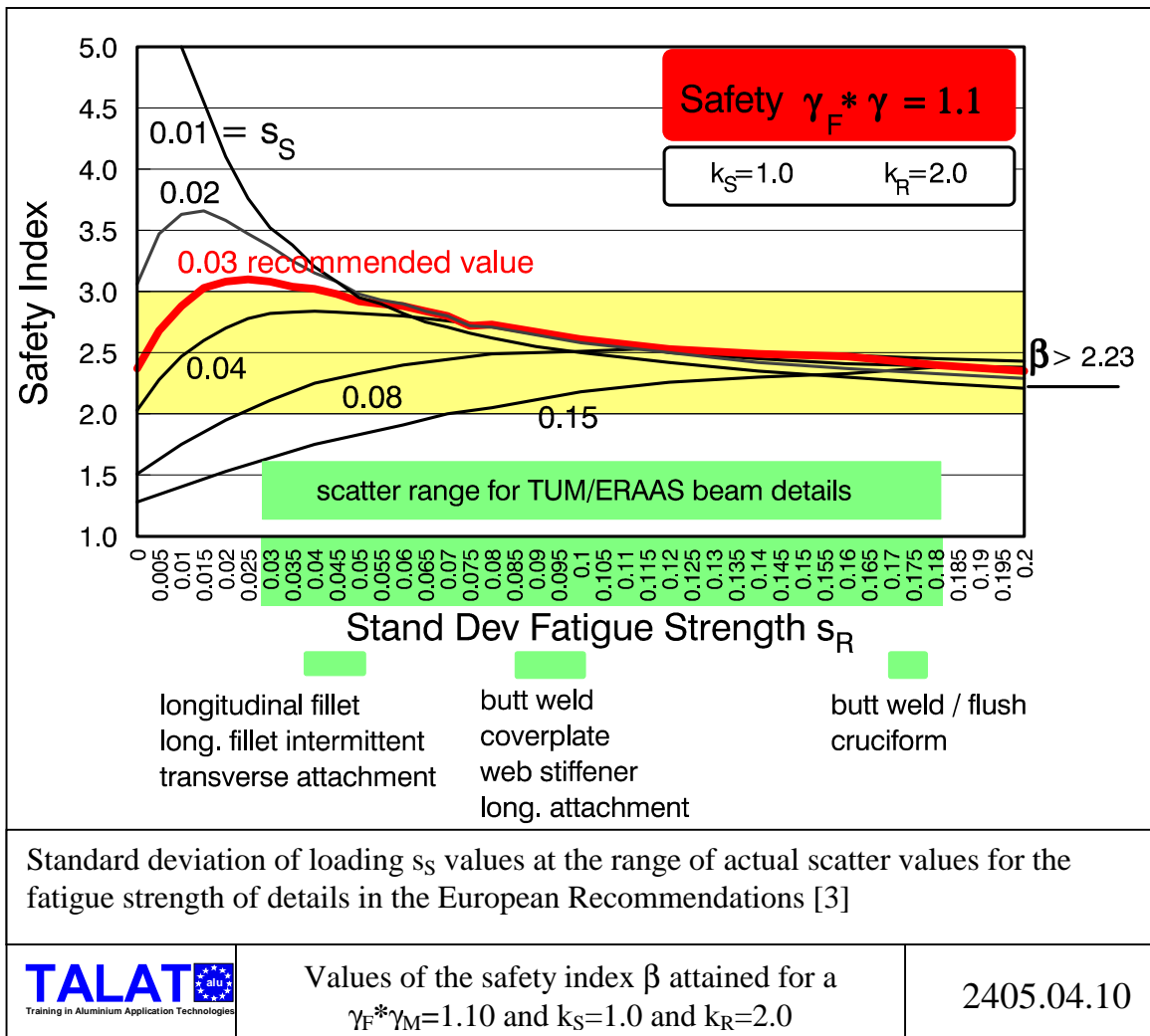
In this and the following formulae k_S corresponds to the k_F value of the spectrum load intensity distribution, as mentioned in the text above, for instance in Table 1.

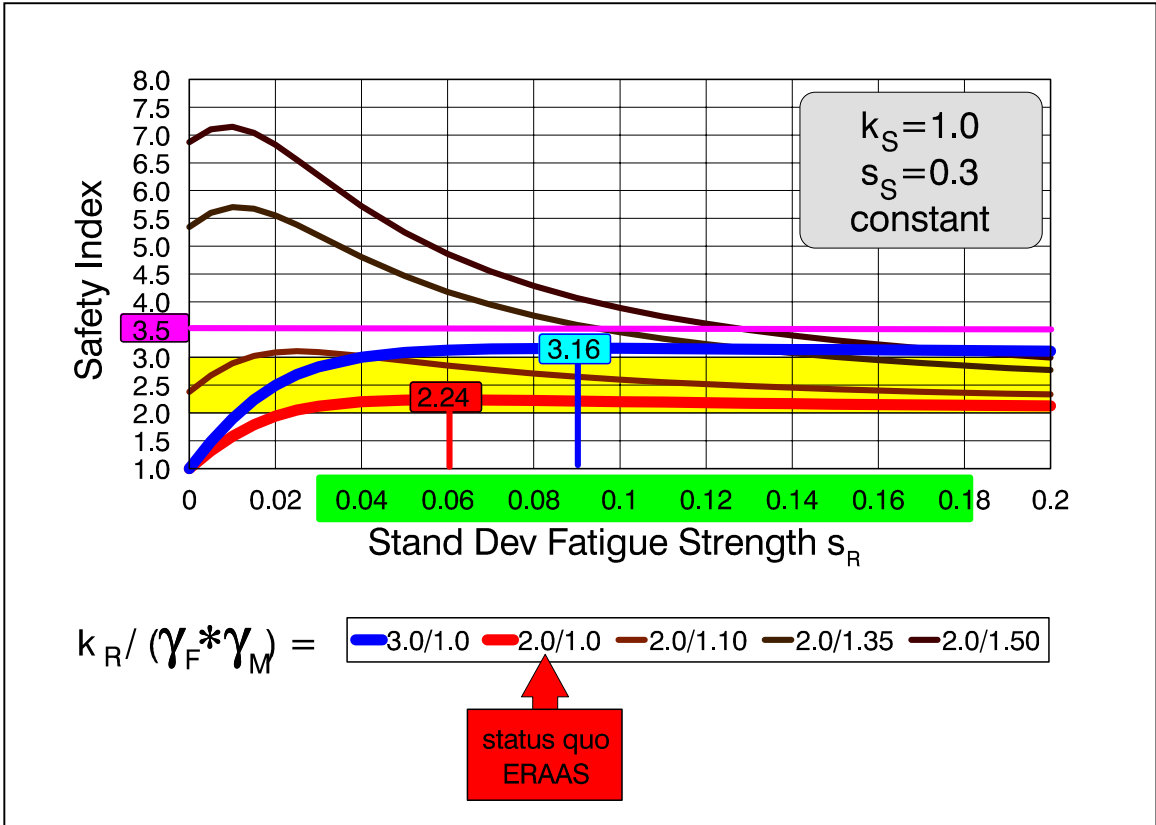
The relationship between these parameters, the partial safety factors and the standard deviations for loading and strength, are demonstrated in the next **Figure 2405.04.09**.



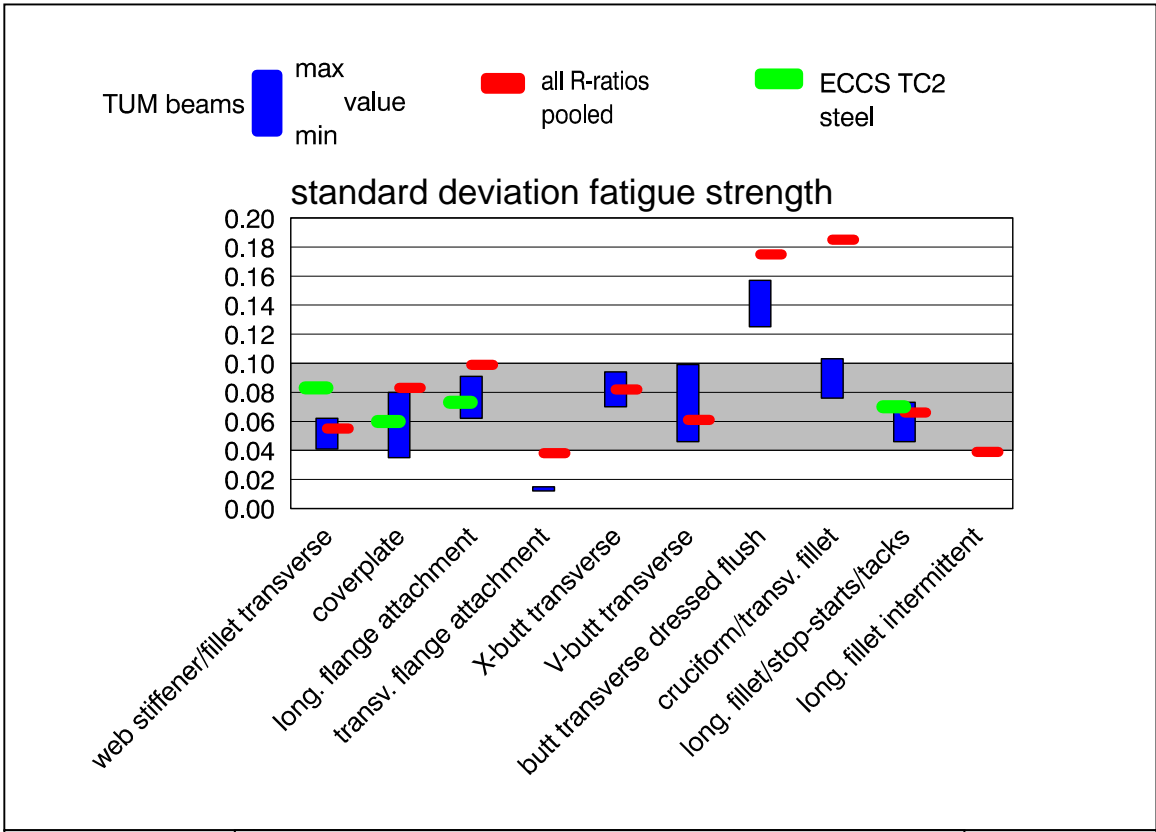
2405.04.03 Safety Index in Aluminium Recommendations

With the above relationships we can now calculate values for the safety index β at actual values of load scatter and for specific values of the partial safety factors. Practically in all cases we assume the values $k_S=1.0$ and $k_R=2.0$ according to the suggested values of CEN/Eurocode being also identical to the assumptions in the European Recommendations for aluminium structures [3]. **Figure 2405.04.10** demonstrates the fact that a value $\beta=3$, as recommended in some documents, can be attained with the recommended $s_S=0.03$ at relatively low scatter in fatigue strength, which has been observed for some details like longitudinal fillet welds or transverse attachments on the flange of beams, but will only reach values around 2.5 for higher scatter in strength. A similar message is conveyed by the diagram in **Figure 2405.04.11**. If the partial safety factors γ_F and γ_M are equal to 1.00 and $k_R=2$, the lowest curve represents the current situation in ERAAS with $\max\beta=2.24$. Only if the design values are defined as mean minus 3 standard deviations, that is $k_R=3$, a value $\beta \cong 3$ can be reached for the range of possible s_R values from 0.03 to 0.18 encountered with the structural details in ERAAS. Even higher β -values, for instance 3.5 as recommended in [5], are possible for $s_R < 0.1$ but only if the product of the partial safety factors $\gamma_F \cdot \gamma_M$ reaches values > 1.2 . The full spectrum of scatter in fatigue strength is shown for the ERAAS details in **Figure 2405.04.12**.



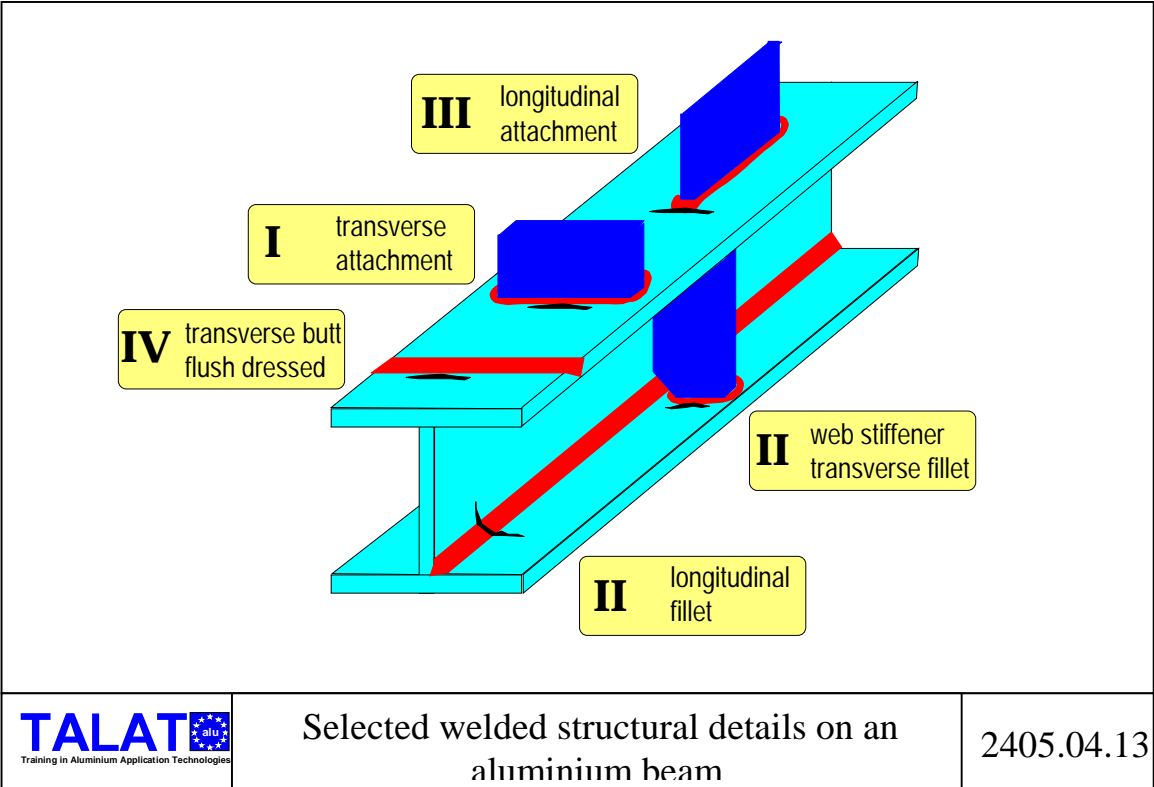



	Influence of design value definition and partial safety factors on the attainable value of the safety index	2405.04.11
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	Values of standard deviation in fatigue strength for ERAAS structural details [3]	2405.04.12
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In the following **Figure 2405.04.13** and **Figure 2405.04.14** the calculated actual β -values are given for a number of characteristic welded structural details. Detail I: transverse attachment with fillet welds on the beam flange non load-carrying exhibits very low values of scatter in strength as has been established through experimental data. Detail II: the longitudinal fillet weld between web and flange, manually welded with stops and starts or tack welds, as well as the web stiffener welded with transverse fillet welds on web and flange. Both have a value of 0.08 in the middle range of scatter values observed. Detail III: the longitudinal attachment welded with fillet welds on the beam flange has a similar value of 0.10, which is also in the middle of the observed value range, but actually represents the upper limit for the greatest number of structural details as demonstrated in **Figure 2405.04.12**. Detail IV: the transverse butt weld with overfill dressed flush, naturally, exhibits a relatively big scatter with 0.18.



scatter in fatigue strength s_R ↓	$\beta = \frac{k_S s_S + \log \gamma_F + \log \gamma_M + 2s_R}{\sqrt{s_R^2 + s_S^2}}$									$\beta = \frac{k_S s_S + \log \gamma_F + \log \gamma_M + 3s_R}{\sqrt{s_R^2 + s_S^2}}$					
0.03	2.12	1.53	1.37	3.10	1.93	1.64	5.20	2.78	2.22	2.83	1.82	1.57	5.90	3.07	2.42
0.08	2.22	2.03	1.82	2.71	2.35	2.07	3.75	3.04	2.59	3.16	2.65	2.29	4.69	3.67	3.06
0.10	2.20	2.12	1.94	2.60	2.41	2.17	3.45	3.04	2.66	3.16	2.83	2.50	4.41	3.75	3.22
0.18	2.14	2.23	2.18	2.04	2.43	2.35	2.85	2.87	2.73	3.12	3.11	2.94	3.84	3.74	3.50
scatter in loading $s_S \Rightarrow$	0.03	0.10	0.15	0.03	0.10	0.15	0.03	0.10	0.15	0.03	0.10	0.15	0.03	0.10	0.15
partial safety $\gamma_F^* \gamma_R \Rightarrow$	1.00			1.10			1.35			1.00			1.35		
The lightly shaded areas indicate values assumed in the current recommendations. The darker areas include β -values higher than 3.5 and point out the necessary level of partial safety values															
	Attainable values of the safety index β .											2405.04.14			

2405.04.04 Summary and Conclusions

Comparing the β values to one another and for the different standard deviations of the various details we find that

- for practical values of loading scatter $s_S = 0.02$ to 0.06 the safety index β reaches its maximum value,
- in case of the ERAAS with $\gamma_F = \gamma_M = 1.00$ / $k_R = 2$ / $k_S = 1$ and mean $s_R = 0.07$ the maximum β -value is 2.236 (for arbitrary loading and resistance $\min \beta = 1.60$,
- a demand for very low scatter in fatigue strength is not per se a guarantee for higher β -values, in several cases depending on the interrelation with the other parameters it may even lead again to somewhat lower values,
- it is much more effective to have reliable information about the loading distribution and a not so high scatter value there,
- in the case of ERAAS (which is also the case for the steel design recommendations as well) for practical values of resistance scatter between 0.03 and 0.18 or for loading between 0.02 and 0.06 the safety index goes not beyond ≈ 2.2 ,
- only in cases with partial safety factors $\gamma_F^* \gamma_R > 1.35$ may values of $\beta \approx 3.5$ be reached, and this only at rather low load distribution scatter,
- the β -value may be enhanced significantly by lowering the fractile of fatigue strength or assuming a lower design value, as demonstrated by values for $k_R = 3$ (this corresponds to a fractile of approximately 99% probability of survival for a sample size of 10 and a confidence level of 0.75),
- it does not appear appropriate though to try to attain higher β -values through magnification of k_R , i.e. lower fractiles of strength or lower design values,

- in the BS 8118 document design values are in a number of cases lower than ERAAS, but unless the actual mean values and the standard deviation values are reported a direct comparison will not be possible,
- neither ERAAS nor BS 8118 assume a priori partial safety factors other than 1,
- the BS 8118 leaves an option for partial safety factors >1 „under certain circumstances“ and as mentioned the ENV 1999 document relates the partial safety factor for loading to certain basic conditions in the estimation of load spectra,
- these may be a) damage tolerant or redundant structures, b) satisfactory degree of inspectability of structural components and their details, easy and not so costly repair, d) reliability of environmental conditions and, especially, loading assumptions during the projected lifetime of the structure, and consequently may be accounted for by the adoption of appropriate partial safety factors.

2405.04.05 References

1. Eurocode 3: Design of steel structures. Part 1.1: General rules and rules for buildings. European Prestandard ENV 1993-1-1, February 1992.
2. DIN 18800: Stahl im Hochbau, Teil 1: Konstruktion und Bemessung, Ausg. November 1990
3. European Recommendations for Aluminium Alloy Structures Fatigue Design, ECCS Doc. No. 68, First Edition, 1992.
4. BS 8118:1991 - Part 1: Code of practice for design. Part 2: Specification for materials, workmanship and protection.
5. Recommendations for the Fatigue Design of Steel Structures, ECCS Doc. No. 43, First Edition, 1985.

2405.05 Unclassified Details


2405.05.01 Design by Reference to Published Data


In the case of unclassified details (§ 5.2.2 of the ENV 1999), i.e. details not covered by the detail categories described within the ENV 1999, these should be assessed by reference to published data where available. One such source may be the compilation of the Aluminium Data Bank (AlDaBa). This data had its origin in the so called CAFDEE committee of the eighties, and it has been maintained at the Technische Universität München - Light Metals and Fatigue Section and at the Iowa State University, Ames, Iowa. It was then enhanced by the data documented and evaluated for the purpose of the European Recommendations for Aluminium Alloys Structures in Fatigue Design (ERAAS Fatigue, 1992), which eventually formed the basis for the ENV 1999, as well - in this context see also IIW Doc. No. XIII-1588-95 „Background Document to Fatigue Design Curves for Welded Aluminium Components“ by R. Jaccard, D. Kosteas, R. Ondra. The Aluminium Data Bank provides also the common platform for the statistical and regressional evaluation of data. These procedures should be observed whenever new data is generated for the purpose of establishing fatigue design lines for new structural components and details not covered in the standard - in this context see next chapter 2405.06 for more details.


2405.05.02 The Aluminium Data Bank (AlDaBa)


In the following pages a brief presentation of the main items of the „AlDaBa“, the Aluminium Data Bank, is given. The AlDaBa was developed and is being run jointly by the Technical University of Munich / Section of Light Metal Structures and Fatigue, Prof. D. Kosteas (phone: +49 89 289 22521, fax: +49 89 289 22522, e-mail: kosteas@lrz.tum.de) and the Iowa State University, Dept. of Civil Engineering, Prof. W.W. Sanders, Jr. (phone: 001 515 294 6048, fax: 001 515 294 8216).


Literature Reference	FW1700.0	FW1700.0
Sander's Literature #:		
Author 1:	Kosteas, D.	
Author 2:	Poalas, K.	
Author 3:		
Author 4:		
Author 5:		
Title:	Voraussage des Ermüdungsverhaltens geschweisster Aluminiumbauteile.	
Publication:	Lehrstuhl für Stahlbau, Technical University of Munich, West Germany.	
Date:	30.06.1986	
Primary Material Tested:	7020	
Secondary Material	5083	
Primary Key Word:	Beams	
Secondary Key Word:	Components	
Other Key Words:	Residual Stresses	
Confidential:		
Library Call #:		











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
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 Abbruch

Welding Process and Procedure		Enable Picklist	A8060.0
Welding Process:	GMAW		
Welding Procedure:	1		
Welding Position:	H		
Filler Material (AA Designation):	5183		
Filler Material (Product Name):	S- AlMg4.5Mn		
Welding Wire Diameter [mm]:	1.6		
Shielding Gas:	1		
Gas Flow Rate [l/min]:	17		
Number of Passes:	1		
Welding Speed [cm/min]:	50		
Welding Voltage [V]:	24		
Welding Current [A]:	280		
Non destruct. Exam. Technique:	N		
Discontinuities:	N		
Pre-weld Treatment:	N		
Post-weld Treatment I:	1		
Post-weld Treatment II:	N		
Post-weld Treatment III:	N		

 Next...

 Prev...

 Save...

 Abbruch

Set ID: **A8055.0** Alloy: **AlMnZ** R-Ratio: **-1.00** **Search** **A8055.0**

Stress	Cycles	Crack	Stress	Cycles	Crack	Stress	Cycles	Crack
178.00	40000.00	0	123.00	147640.00	0			
178.00	40000.00	0	123.00	147640.00	0			
178.00	50140.00	0	123.00	147640.00	0			
178.00	59550.00	0	123.00	166230.00	0			
178.00	59550.00	0	62.00	1064220.00	0			
178.00	59550.00	0	62.00	1457050.00	0			
123.00	89380.00	0	62.00	2569860.00	0			
123.00	89380.00	0	62.00	2923420.00	0			
123.00	89380.00	0	62.00	3163310.00	0			
123.00	89380.00	0	62.00	3163310.00	0			
123.00	89380.00	0	62.00	3163310.00	0			
123.00	89380.00	0	62.00	2814860.00	0			
123.00	89380.00	0	62.00	596850.00	0			
123.00	89380.00	0	62.00	596850.00	0			
123.00	88400.00	0	49.00	706210.00	0			
123.00	128350.00	0	62.00	572000.00	0			
123.00	44160.00	0	62.00	575750.00	0			
123.00	147640.00	0	62.00	706180.00	0			
123.00	147640.00	0						

Testpoints #: **37**
Stress Levels: **5**

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 Abbruch

Data Processing: Analysis Type and Analysis Options:
Analysis: **Linear Regression**; Option: **All points were used for analysis.**

Average Values:
Mean Log[Stress]: **1.92** Mean Log[Cycles]: **5.58**

Equation of Regression Line:
$$\text{LogN} = -2.13 * \text{LogS} + 9.68.$$

Variance and Standard Deviations:
SSR: **0.03** Std. Dev. Log S: **0.06**
Variance of Log N: **0.02** Std. Dev. Slope: **0.49**
Std. Dev. Log N: **0.12** Std. Dev. Intercept: **0.94**

95 % Confidential Intervals for Slope and Intercept Parameters:
 $-6.27 < \text{Slope} < 2.00$ $1.72 < \text{Intercept} < 17.63$

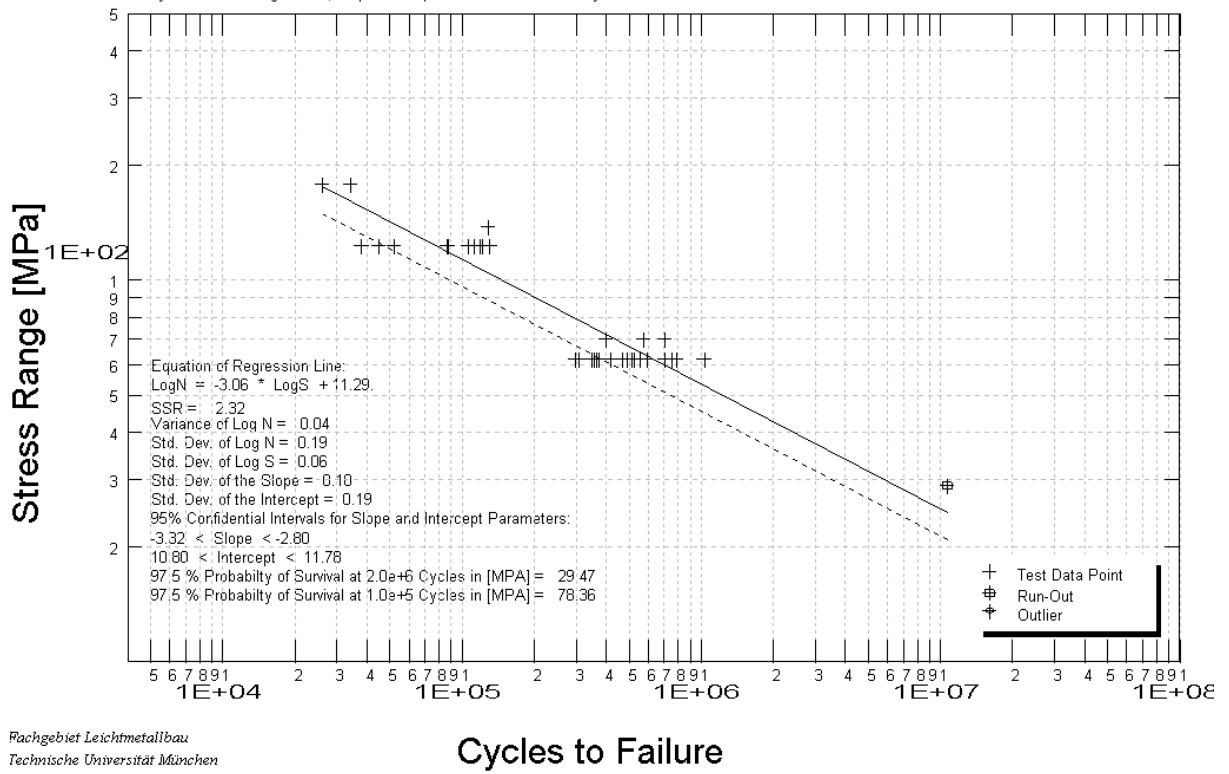
Estimated Values:
Estimated Mean Stress at 2E+06 Cycles To Failure: [MPa] **38.12**
Estimated Mean Stress at 1E+05 Cycles To Failure: [MPa] **155.12**
Estimated LogN Stress Range at 30 MPa: **6.52** Cycles: **3334328**
Estimated LogN Stress Range at 50 MPa: **6.05** Cycles: **1120694**
Estimated LogN Stress Range at 100 MPa: **5.41** Cycles: **255246**

Probability of Survival:
97.5 % Probability of Survival at 2E+06 Cycles in [MPa]: **12.20**
97.5 % Probability of Survival at 1E+05 Cycles in [MPa]: **49.64**

OK Print



Analysis: Linear Regression; Option: All points were used for analysis.



Fachgebiet Leichtmetallbau
 Technische Universität München

2405.06 Testing for Fatigue Design

The second option to deal with unclassified details is by conducting fatigue acceptance tests in accordance to Annex C.3 of the ENV 1999. In doing this the provisions of Annexes C.1 on the derivation of loading data and C.2 on the derivation of stress data at critical locations of the structural component studied should be observed.

Annex C.3 handles the derivation of endurance data, i.e. the estimation of the respective design S-N curve. With known or estimated stress history and spectrum data specimens can be manufactured keeping (and fully documenting) the same dimensions and procedures as intended to be used in the final design. Any NDE and acceptance criteria should also be documented. Loads and stresses should be recorded during the test by means of one or more strain gages at critical locations (in appropriate distance from notches, weld toes for instance, in order to record „nominal stresses“). A sample size of at least 8 specimens is required for the range between 10^4 and 10^7 cycles - see here also recommendations under 2405.01.02 or 2405.03.

In the double logarithmic S-N diagram ($\log\Delta\sigma$ - $\log N$) a mean curve shall be estimated and a design curve, parallel to the mean, shall be obtained, either at a mean minus 2 standard deviations level or not higher than 80% than the mean, whichever is lower.

In case that damage tolerance design is conducted a record of fatigue crack growth with cycles should be obtained.

Similar conditions should be observed in testing full-size structural components.

When the structure is expected to give a safe life performance, then design should satisfy the condition that

$$T_m \geq T_L \cdot F$$


where T_L is the design life in cycles

T_m is the mean (endured) life to failure in cycles

F factor defined in the table below and depending on the actual (effective) number of specimens available

results of tests	sample size = no. of specimens tested							
	1	2	4	6	8	9	10	100
Identical specimens all tested to failure. All specimens failed.	3,80	3,12	2,73	2,55	2,48	2,44	2,40	2,09
Identical specimens all tested <i>simultaneously</i> . First sample to fail.	3,80	2,67	2,01	1,75	1,60	1,54	1,54	0,91

Population standard deviation assumed as log 0,176

	Fatigue test factor F	2405.06.01
---	-----------------------	------------

When the structure is expected to satisfy damage tolerant design then the failure criterion will be dependent upon the life of a crack reaching a size which could be detected by a method of inspection which can be applied in service. It also depends on the crack growth rate, critical crack length considerations, and the implications for the residual safety of the structure and the costs of repair.

2405.07 Damage Tolerant Design


This chapter is divided into the following parts:

- In a general introductory part from *{TALAT 2403.03 Principles of Fracture Mechanics}* which gives the basic information about the fracture mechanics concept and the assumptions and analytical expressions governing crack geometry /stress relationships - this may be regarded as an informative part; it may be omitted in a first reading, contains information on the basic definitions, though.
- More application oriented is the material from *{TALAT 2403.05 Fracture Mechanics Instruments for Structural Detail Evaluation and TALAT 2403.06 Calculation of an Example}* which deals with proposals for treating practical cases, and gives actually two life calculation examples.
- The last part of this chapter presents an outline of the provisions of the *ENV 1999-2, 2.3 and Annex B* on „damage tolerant design“ on the basis of fracture mechanics concepts. It gives practically the procedure steps for carrying out the calculations.

Note: Further introductory information to the fatigue crack initiation and propagation characteristics may be taken from chapter 9.9 Strain-Life Approach and the respective parts from *{TALAT 2401.02 and 2401.04 and 2401.05}*.

An excellent overview of fracture mechanics applications and the fatigue crack propagation in aluminium alloys along with life estimation calculations is presented in the paper by Dr. R. Jaccard „Zum Bruchverhalten von Aluminiumbauteilen“ published in the STAHLBAU Spezial „Aluminium in Practice“, 67(1998), Ernst & Sohn/Wiley, Berlin, pp 54-65.

2405.07.01 Outline of Provisions in ENV 1999-2, 2.3 and Annex B

<p>Safe Life - S/N design curves (based on component tests) <i>no cracks tolerated or assumed</i></p> <p>Damage Tolerance - based on fracture mechanics calculations <i>cracks assumed</i></p> <p>Design by Testing - in lieu of standard data</p>		
	Fatigue design methods	2405.07.01

<p><u>Drawings</u> - full details susceptible to fatigue, required fatigue class</p> <p><u>Manufacturing Specification</u></p> <p><u>Operation Manual</u> - assumed loads and design life, repair methods</p> <p><u>Maintenance Manual</u> - methods, locations and frequency of inspections, max permissible crack, details of acceptable repair methods</p>		
	Design documents required by ENV 1999	2405.07.02

if $D_d = \sum \frac{n_i}{N_i} > 1$ i.e.

if safe life $T_s < design\ life\ T_d$

i.e. $\frac{T_d}{T_s} = D_d > 1$

for all initiation sites on or close to surface, accessible in service, unless crack arrest possible or redundancy in structure available

and where no alternative action such as

- redesign of detail to reduce stress or

- change the detail to one with higher category

is undertaken



Prerequisites for damage tolerant design

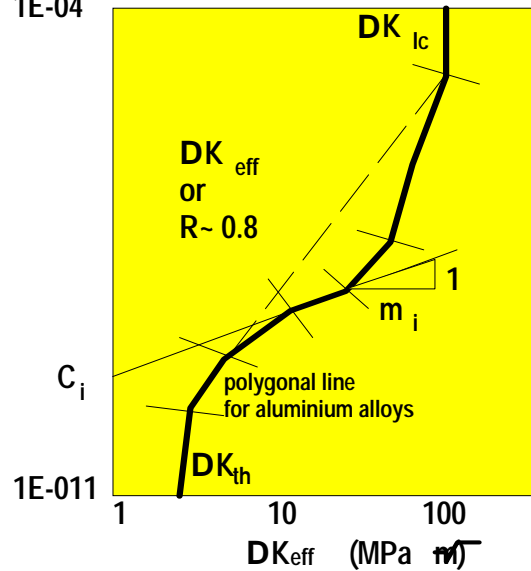
2405.07.03

critical fracture length l_f depends on the "fracture criterion"

assumed minimum detectable crack size l_d in mm

	inspection method	
	visual/ magn. aid	liquid penetr. aid
plain+smooth surface	20	5
rough weld cap	30	10
sharp corner weld toe	50	15

da/dN (m/cycle)
1E-04



Parameter values for damage tolerant design

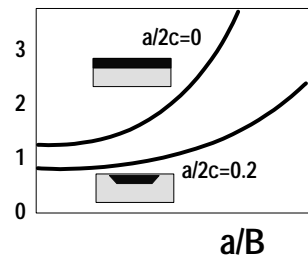
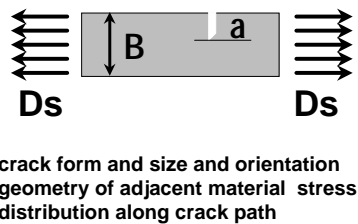
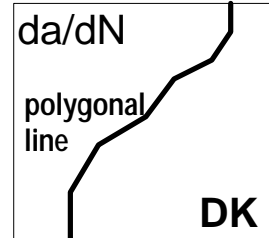
2405.07.04

Cracks → Propagation → Life to Failure

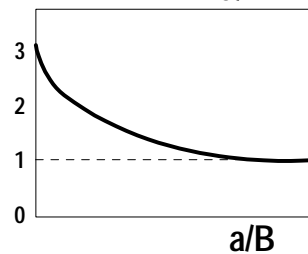
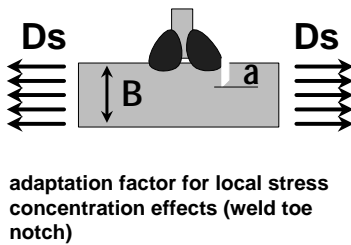
$$da/dN = C * [Ds * \sqrt{a} * f(y)]^m$$

$$da/dN = C * DK^m$$

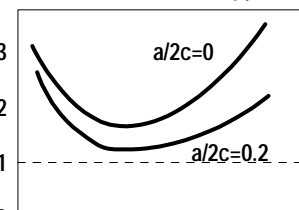
slope m and C variable section-wise



Global geometry factor
 $Y = f(y)\pi^{-0.5}$

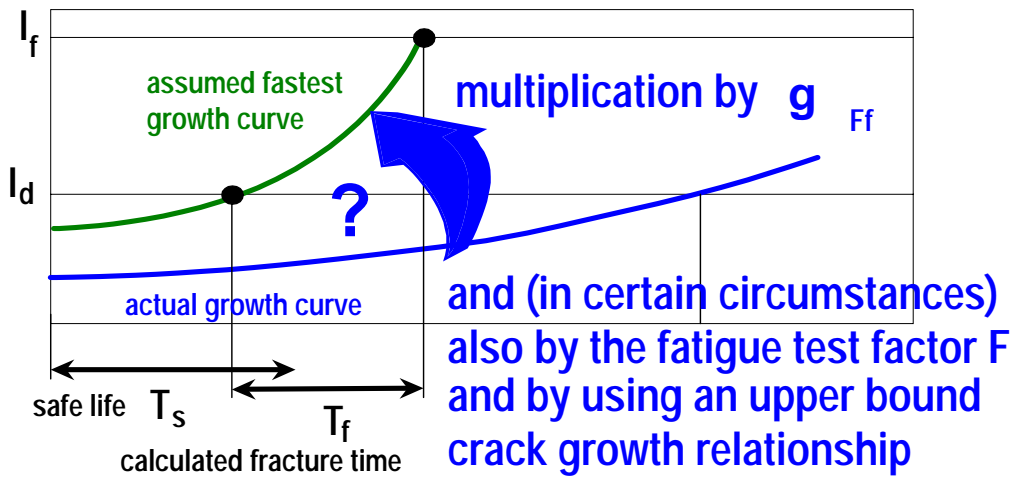
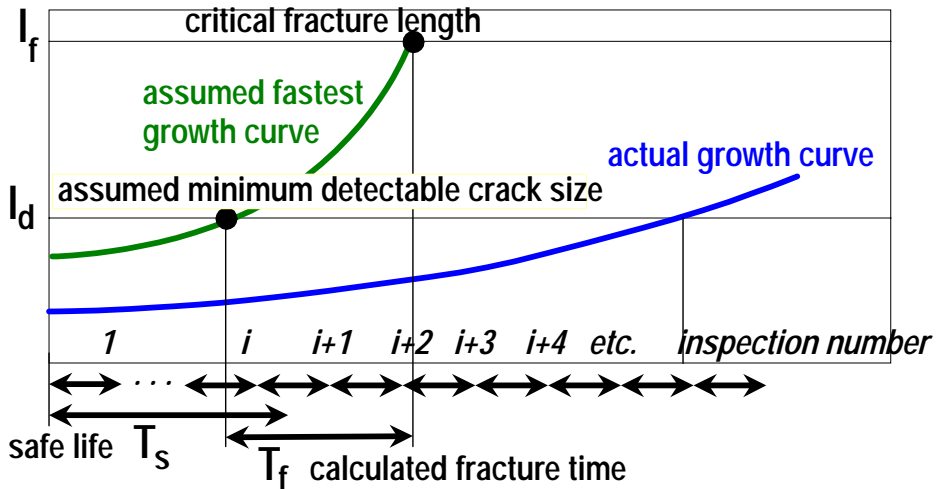


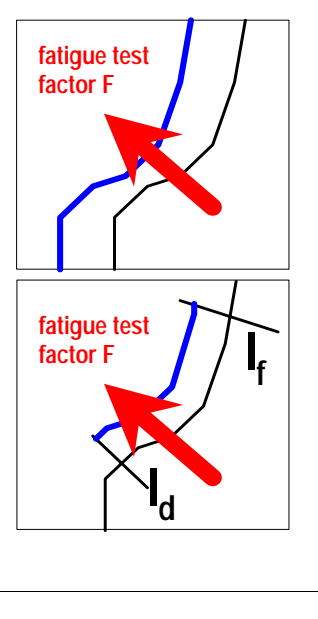

Local geometry factor M_l

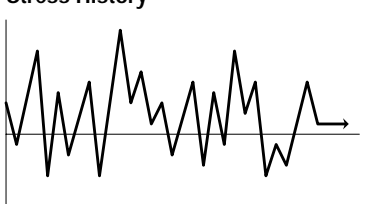
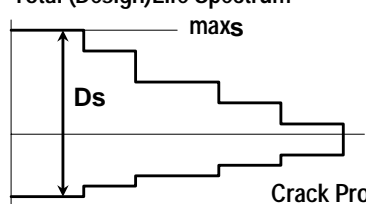
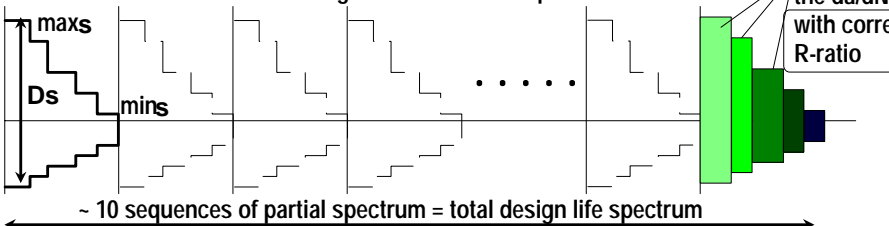



Global and local effect

- ✓ first inspection before safe life elapses !
 - ✓ subsequent inspections at regular intervals ✓
- when measured $l > l_d$ fitness-for-purpose assessment and possible repair



by calculation	and/or	by testing
$T_f = \int_{N_d}^{N_f} dN = \int_{l_d}^{l_f} \frac{da}{C_i \Delta K_{i,eff}^{m_i} f(g)}$ <p>l_f = is the critical fracture crack length l_d = is the assumed min safe value of detectable crack length</p> $\Delta K_{i,eff} = \gamma_{Ff} \Delta \sigma \sqrt{a} f(g)$ <p>m_i and C_i are the respective parameters of the da/dN – polygon</p> <p>the inspection intervall is</p> $T_i \leq 0.5T_f$	<p>standard test specimens - same material as in crack path</p> <p>component tests correct materials, geometry and manufacture with relevant applied force pattern</p>	
	<p>Calculation of inspection intervals in damage tolerant design</p>	<p>2405.07.09</p>

<p>① Stress History</p>  <p>② Total (Design)Life Spectrum</p>  <p>③ Partial Spectrum = 1/10 of the Total Spectrum same D_s - same R - descending order of stress amplitudes</p>  <p>Crack Propagation by Cycle Integration for each Block with $D_s = \text{const.}$ through the da/dN-Polygon with corresponding R-ratio</p> <p>~ 10 sequences of partial spectrum = total design life spectrum</p>		
	<p>Integration of fatigue crack growth - accounting for the loading spectrum</p>	<p>2405.07.10</p>

1

overall
final
fracture

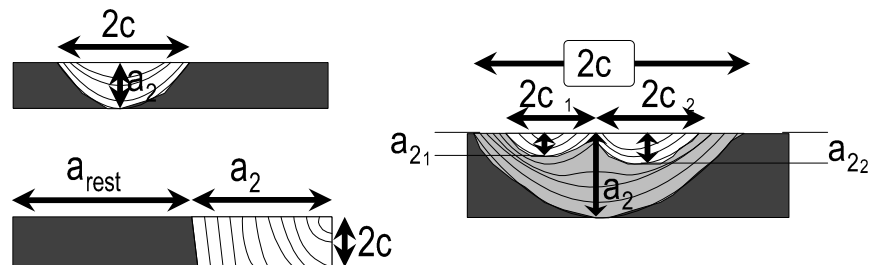
$$\frac{f_a}{\gamma_{M2}} = \sigma_{all} = \sigma_{eff} = \frac{\gamma_F F_k}{A_{critical}^{rest}}$$

$$A_{critical}^{rest} \rightarrow a_{rest} \rightarrow l_f$$

see examples on following diagrams

simplifying as in ex. "b" at
critical fracture stadium with
plate thickness $t = 2c$

$$l_f = a_f = a_{tot} - a_{rest} = a_{tot} - \frac{\gamma_F F_k \gamma_{M2}}{f_a t}$$



Calculation of the critical fracture length l_f depending on the fracture criterion (final critical crack size such as „through thickness crack“ or reaching the critical cross section with ultimate strength, etc.) - F_k : Load [N] - γ_F : partial safety factor for loading in ENV 1999-1, 2.2, 2.3 - γ_{M2} : partial safety factor for resistance in ENV 1999-1, 5.1.1 equal to 1,25 - f_a : limit value for resistance, local in net cross section, ENV 1999-1, 5.3.5 or $f_a = f_u$ = characteristic value of ultimate stress of the aluminium alloy as in Table 3.2a-d or 3.3 [N/mm²]

2

final damage in complicated crack propagation path(s)
or
partial damage affecting serviceability

- estimation of critical fracture crack length l_f will have to be based upon
- computation of possible crack propagation path(s) by integration of the da/dN line for a critical crack length a_{rest} corresponding to a critical cross section A_{rest} (iterative procedure)
 - definition of l_f according to the overall geometry
see example "d" on following diagrams



Calculation of the critical fracture length l_f in complicated crack paths

2405.07.12

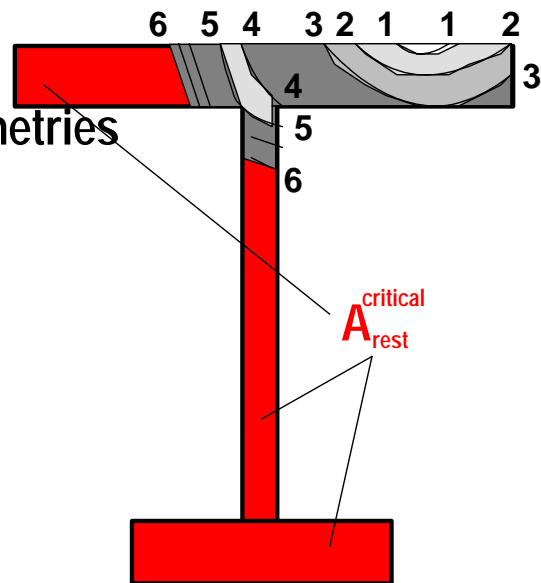
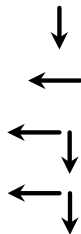
successive different geometries and crack propagation directions

from 1 to 2 to 3

from 3 to 4

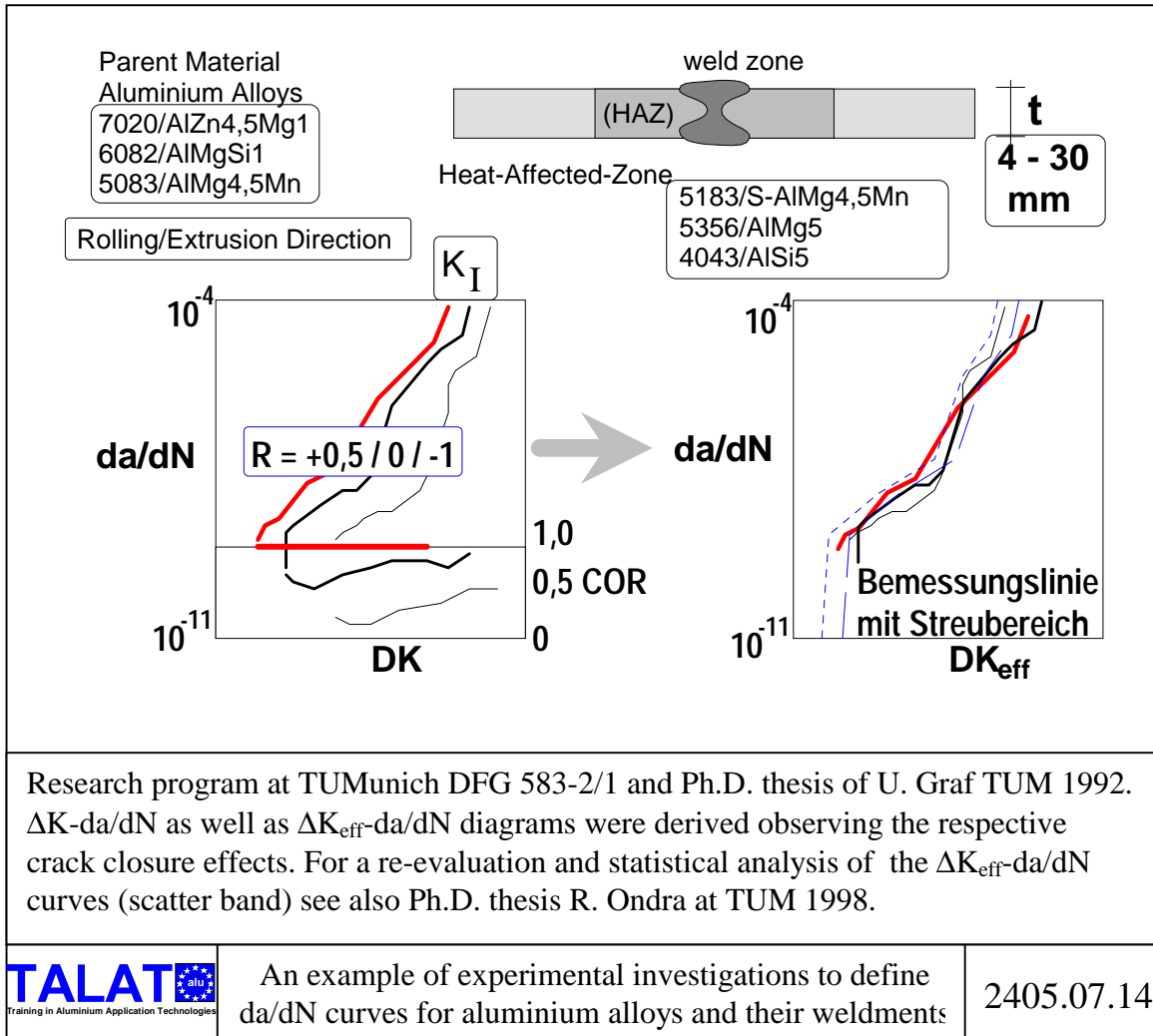
from 4 to 5

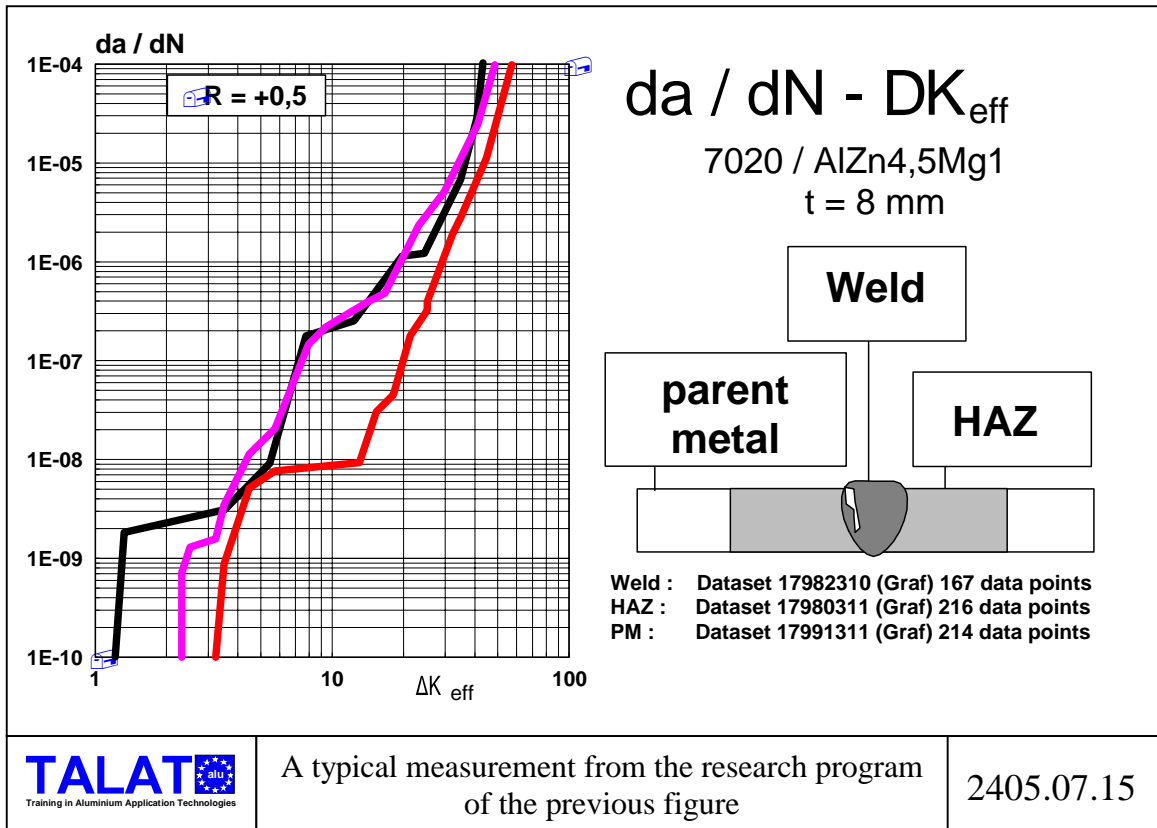
from 5 to 6



Successive crack geometries and Crack propagation directions

2405.07.13





2405.07.02 Summarizing the Life Prediction Procedure

Fatigue life prediction based on fracture mechanics still requires the following engineering assumptions: (1) initial crack length, (2) residual stress distribution, and (3) fatigue crack growth of short cracks.

Initial crack length is determined by scanning electron fractography and/or fatigue crack calibration.

Residual stress is accounted for and short crack growth is approximated by the $K_{\max}=\text{const}$ fatigue crack growth relation.

Fatigue life is calculated by using a single phase of fatigue crack growth from the initial crack length (usually a short crack) to the final crack length (plate thickness or any other critical dimension) and the conservative combination of $K_{\max}=\text{const}$ and $R(K_{\min}/K_{\max})$ curve of the fatigue crack growth data.

Single slope fatigue crack propagation data is not recommended.

Fatigue crack propagation data is given in ENV 1999-2 as a conservative envelope of measured values of common aluminium alloys - further upper boundary estimates for crack propagation and scatter data are to be found in the paper by Kosteas and Ondra in the STAHLBAU Special Issue on „Aluminium in Practice“.

Fatigue calibration is an engineering tool to allow the application of fracture mechanics at the early stage of fatigue crack growth and to estimate the damage caused by a load spectrum.

The calibrated initial crack length is influenced by the stress field parameters, the selected fracture mechanics model and the fatigue crack propagation. If all parameters are in agreement with the S-N test conditions and specimen properties, the calibrated initial crack length is identical to the physical crack length.

The fatigue crack propagation is the relevant parameter of the initial crack length calibration. The measurement of the fatigue crack propagation is in reality nothing else than a special S-N test of a specimen with the worst possible notch condition, a fatigue crack, and a special loading condition. Due to the computer controlled loading and the severe (reproducible) notch condition the fatigue crack propagation data show less scatter than respective S-N data.

Conservative fatigue crack propagation data lead to shorter initial crack length.

The S-N simulation and the initial crack length calibration must be performed using identical fatigue crack propagation data and fracture mechanics models.

The calculation of cumulative damage due to fatigue can be performed using a fracture mechanics evaluation of the fatigue crack growth including the early stage of fatigue crack growth.

2405.08 Sequence Effects

Within this lecture series only brief information is presented in the following pages from the material in *{TALAT 2401.02}*. Further details may be taken from the respective general literature mentioned on the front page.

2405.09 Strain-Life Approach

The information in the following pages is understood as supplementary information to various chapters of this lecture no. 9. It is optional, and actually supplements also the mentioning of the strain-life concepts presented under lecture no. 2. It is taken from the material in *{TALAT 2401.02 Fatigue Damage and Influencing Parameters}* and *{TALAT 2401.05 Local Stress Concepts and Fatigue}*.

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